DOCUMENT RESUME

ED 389 564 SE 057 207

AUTHOR Price, Terry M.; Adams, Verna M.
TITLE A Theory of Second-Generation Constructions.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE Oct 95
CONTRACT RED-9254922
NOTE 8p.; Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (17th, Columbus, OH, October 21-24, 1995). For entire conference proceedings, see SE 057 177.
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Classroom Communication; *Concept Formation; *Discussion (Teaching Technique); *Division; Grade 7; Junior High Schools; Junior High School Students; Mathematics Instruction
IDENTIFIERS *Second Generation Constructions

ABSTRACT

When a student begins to appropriate an idea from classroom discourse, the idea is likely to be perceived incompletely because the speaker's understanding of the idea cannot be conveyed in its entirety through the discourse. Under the guidance of the teacher, the discourse serves to stimulate further development of the idea itself, the development of connections to existing knowledge, and its use in constructing new content. The way in which students appropriate ideas presented to them by another individual and make them their own are called "second-generation constructions." In this study, all students were creating second generation constructions, including the student who originally presented an idea to the class, because he acquired the divisibility by 8 rule from his father. Implications for instruction include the importance of revisiting ideas from time to time to increase students' perception of their value and encouraging discourse to facilitate the constructive process. (Author/SW)

A Theory of Second-Generation Constructions

Terry M. Price and Verna M. Adams

Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education

(17th PME-NA, Columbus, OH, October 21-24, 1995)

A THEORY OF SECOND-GENERATION CONSTRUCTIONS

Terry M. Price, Washington State University
Verna M. Adams, Washington State University

When a student begins to appropriate an idea from the classroom discourse, the idea is likely to be perceived incompletely because the speaker's understanding of the idea cannot be conveyed in its entirety through the discourse. Under the guidance of the teacher, the discourse serves to stimulate further development of the idea itself, the development of connections to existing knowledge, and its use in constructing new content. The way in which students appropriate ideas presented to them by another individual and make them their own are what we call *second-generation constructions.* In this study, all students, including the student who presented the idea to the class because he acquired the divisibility by 8 rule from his father, were creating second generation constructions. The theory we propose is a substantive theory (Glaser & Strauss, 1967; Glaser, 1978), not a formal theory.

In this study, we attempt to describe students' constructions of mathematics in a seventh-grade mathematics class as they talk about dividing by 8. The focus of the investigation is not students' constructions in a teaching experiment. Instead, we focus on the "everyday activity" (Lave, 1988) in the practice of doing mathematics in a class taught by the "regular" teacher. We look at the ways in which students appropriate an idea presented to them by another individual and make it their own. Because context is integral to the cognitive events involved in constructions (Rogoff, 1984), the phenomenon is likely to have important characteristics related to the context.

The data discussed in this paper was collected during a unit on number theory and is part of a larger study focused on the relationship between classroom discourse and problem solving. Because the idea was presented by a student but did not originate with him, the understandings and connections he developed are what we call *second-generation constructions*; that is, second-generation constructions occur when a student appropriates an idea from the discourse and constructs connections to her/his existing knowledge base. The student who presented the rule in this study acquired it from his father.

A student idea is not essential to a theory of second-generation constructions and we do not claim to present a full-blown theory. We focused on a single case that occurred naturally in the classroom as a result of the teacher's decision to promote discussions of student thinking and justification. Although the idea of second-generation constructions emerged from our data (that is, it described the development of a student idea), the idea could have been introduced by the teacher. The importance of student ideas is emphasized by the National Council of Teachers of Mathematics (1991) in their description of the teacher's role in discourse. We believe an advantage to following the development of a student idea is increased student ownership of the content.

Preparation of this paper was supported in part by National Science Foundation, Grant No. RED-9254922. Any opinions, conclusions, or recommendations are those of the authors and do not necessarily reflect the views of the National Science Foundation.

The Study

Data collection consisted of a combination of classroom observations (video tapes, audio tapes and field notes), whole-class surveys, and interviews. We draw on the grounded theory method of Glaser and Strauss (1967) and Glaser (1978) in which, after identifying a phenomena of interest from the data, additional data is collected and coded. Coding of data began as field notes were taken and continued through the analysis. Categories began to emerge during the analysis through incident-to-incident comparisons. The analysis progressed to comparisons of incident to properties of a category. As new categories emerged, subsequent data collection was influenced by the results of the previous data. Analysis continued after data collection was completed to further develop the properties of the categories. Finally, data from all sources was coordinated and sorted into groups based on students' responses to the final survey.

A Description of Blayne's Participation

In this section, we describe Blayne, the student who gave the rule to the class. The following timeline shows the amounts of time between data-collection points. Daily observations of the classroom began before the start of this study and continued beyond the time frame of interest to this study.

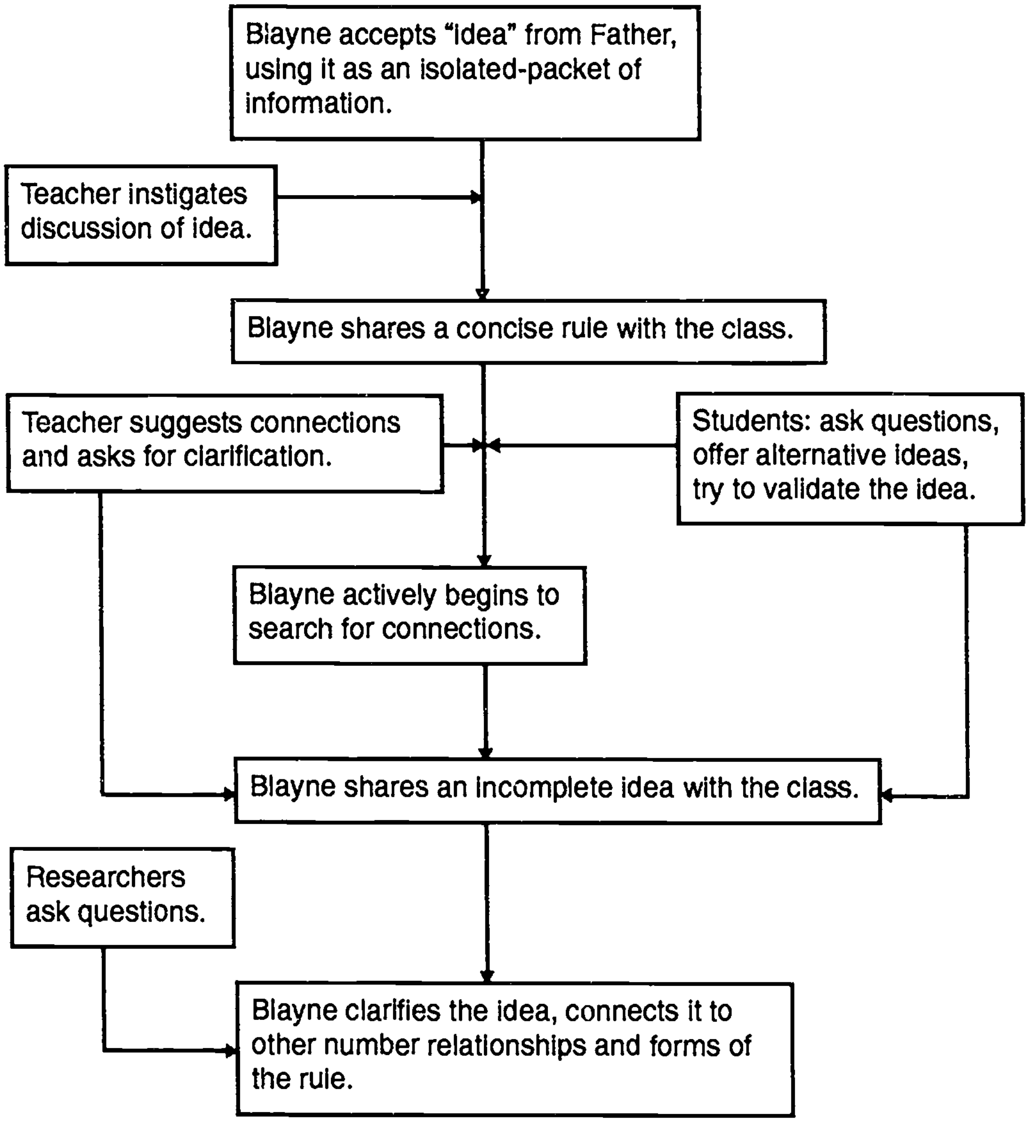
Data Collection Points and Timeline

Initial Presentation	Presentation	Presentation	Initial Class Survey	Blayne's Interview	Second Survey
Sept. 21	Sept. 23	Sept. 26	Sept. 29	Oct. 6	Oct. 25

————> 2 days ————> 1 day ————> 3 days ————> 7 days ————> 19 days

In the interview Blayne explained that when his father told him the rule the year before, he had been trying to determine what numbers would divide into other numbers. His father gave him a little "trick" for 8 where you divide by 2 three times. Blayne was motivated to remember and use the rule. When Blayne shared the rule with the class on September 21, it followed a class discussion of other divisibility rules. In spite of Blayne's familiarity with the "eight rule", his initial statement was garbled: "You divide by 2 six or eight times." Blayne's responses to questions from the discourse suggested that his understanding of the rule was not connected to other knowledge. He had initially treated his father's idea as an isolated-packet of information to be called upon when working divisibility problems. The following diagram shows the structure of what happened.

At each data-collection point, Blayne gave a more concise statement of the rule and what it meant, but not without glitches. On the second day, for example, he began to show on the overhead how the rule applied to the number 56. Before he completed the example, he shifted from a written and oral form of communica-



tion to an oral form only and, at the same time, switched the number to 16. Initially, Blayne indicated that the results of the successive divisions should be "even". By the interview he talked about the division process as not having either a "remainder" or a "decimal" result and he stated that a decimal result after the second division meant that eight would not go into the number. We were surprised that on the initial survey Blayne indicated that the eight rule would not always work, stating his justification: " I think that because in class we tried it and the number that didn't work was 12345678." In the interview, he revealed that he had made an error on the survey. His confusion stemmed from the fact that while in class, he thought that the two- rule gave a whole number answer, but dividing by eight did not. Later he used a calculator and obtained a decimal remainder for both calculations.

On September 23, Blayne clearly did not know that dividing by 2 three times could be related to 2^3. By the time that he responded to the second survey, how-

ever, he had made the connection, not only to the different symbolic representations of 8, but also to different forms of the rule that included: (a) If a number has 3 factors of 2, then it is divisible by 8. (b) If a number is divisible by 8, then you can divide it by 2, divide the answer by 2, and divide that answer by 2 again, getting a whole number answer each time you divide. If any of the answers is not a whole number, then the original number is not divisible by 8. (c) If 2^3 is a factor of a number, then the number is divisible by 8. We believe that the discourse with the teacher, other students, and researchers sustained his attention and oriented it toward number relationships and different forms of the rule.

Blayne's responses on the second survey indicated that he had formed connections to other mathematical knowledge and the rule was no longer just a trick. He had not, however, generalized the rule to division by powers of other numbers. When he was asked to determine if 675 was a multiple of 27 without dividing by 27, he summed the digits on both numbers and divided the results. He, however, was not alone; only three students attempted to generalize the rule to powers of three.

Toward a Theory of Second-Generation Constructions

From our observations of Blayne and other students, we begin to formulate a description of the characteristics of the influence on student constructions (e.g., acceptance-nonacceptance) and the characteristics of the construction process (e.g., connections, type of justification they use) that result from the events in the classroom.

Acceptance/Nonacceptance of the Rule

This characteristic of the influence on student constructions was evident in the discourse by the questions and comments of the students. For the following discussion, acceptance/nonacceptance was determined by the connections that students made to other forms of the rule and to their spontaneous use of the rule on the surveys. That information was then coordinated with the information from the videos and field notes about student participation in class discourse. The students generally fell into the categories of either accepting, exploring, or resistant to the rule, containing 9, 16, and 2 students, respectively.

Acceptance: The 9 "accepting" students revealed their acceptance through spontaneous use of the rule and understandings of other forms of the rule. All of them spontaneously used the rule as justification on the second survey for the question: Suppose you divide a number by 2 and get an even number. Then you divide the answer by 2 and get an odd number. Is the original number a multiple of 8? On the first survey, 5 of these students spontaneously divided by 2 three times when they were asked: Is 2000 a multiple of 8? Furthermore, these students had made connections to other forms of the rule and, for the most part, believed that the rule would always work They relied heavily on *example-based justification*; that is, they used a larger number of examples than other students to convince themselves the rule worked. Their classroom participation was minimal and their

construction process was silent. Their thinking was not evident in the classroom discourse. Generally, when they did offer ideas to the class, the ideas consisted of examples of numbers not divisible by 8 and language clarification.

Exploration: This group of students was less accepting of the rule than the first group. About half of this group spontaneously used the rule in a calculation, but no one spontaneously used the rule as justification. Only 6 out of the 16 students in this group made connections to other forms of the rule. In general, these students were undecided with respect to the rule, but were more inquisitive than other students. They were more actively engaged in the dialogue, offering interpretations of the results of the discussions, exploratory conclusions about the workability of the rule and alternate revisions of the idea. More than half of the students who participated in the classroom discussion fell into this group and were clearly actively trying to construct an understanding of the rule. This group did not make up their minds about the rule as quickly as the other two groups and gave a mixed pattern of responses on the surveys. "Failures of context" (Edwards & Mercer, 1987) in the discourse affected these students more than others.

Resistance: Neither student in this group spontaneously used the rule on the surveys or made connections to other forms of the rule. They thought the rule was time consuming and inefficient. One stated: "I don't understand why you go through the trouble. Why don't you just divide by 8 to begin with?" He had what might be considered a healthy skepticism about proof via examples, stating that he did not believe the rule would always work because "Someone will prove him wrong somehow." The other student considered the rule to be "undependable." This group maintained their resistance to construction, in spite of social interaction, because they valued efficiency. We believe that they could be persuaded to pursue an active construction if given acceptable justification.

Perception of Value

Perception of value had two properties: value attached to the rule and value attached to people. Unlike other students in the class, the two who were resistant to dealing with the rule did not place any value on its use. Considerable value was given to Blayne, and his confidence in his own abilities was affected in a positive manner. The teacher created a positive climate where Blayne felt comfortable expressing his idea. He was perceived by the teacher and some of his peers as having a higher level of understanding than was actual fact, and some students began to perceive themselves as less competent than Blayne. Because of the teacher's perception, Blayne was allowed more "floor time" than other students for the exploration of their ideas. This floor time was significant because he benefited more from the discussions than other students. His idea was given value and, during the investigation, was referred to as a "theory". That language implied that it had importance, perhaps more importance than others.

Discussion

Much of what we want to say is left for further development in a longer paper. In the natural environment of the classroom, the ways in which students appropriate mathematical knowledge from the discourse is a nontrivial process. In general, students acquire mathematical ideas introduced to them by someone else (e.g., the teacher, peers, parents). This appropriation requires precious classroom time and special attention to the discourse. In this study, time allotted to discussion sustained the interest in the idea. In addition, the research itself influenced the perception of value. The role of the perception of value should not be taken lightly. Blayne had a full year to develop ideas related to his rule, which he did not do without the sustained interest of the teacher and other students, their questions and their comments. An implication for planning instruction is that ideas should be revisited over time and the discourse is an important component of the construction process.

References

Edwards, D., & Mercer, N. (1987). *Common knowledge: The development of understanding in the classroom.* New York: Routledge.

Glaser, B. G. (1978). *Theoretical sensitivity.* Mill Valley, CA: Sociology Press.

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research.* New York: Aldine De Gruyter.

Lave, J. (1988). *Cognition in practice.* New York: Cambridge University Press.

National Council of Teachers of Mathematics (1991). *Professional Standards for Teaching Mathematics.* Reston, VA: Author.

Rogoff, B. (1984). Introduction: Thinking and learning in social context. In B. Rogoff & J. Lave (Eds.), *Everyday cognition: Its development in social context* (pp. 1-8). Cambridge, MA: Harvard University Press.